Interacting QFT on spacetimes with horizons

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19 декабря, 2024
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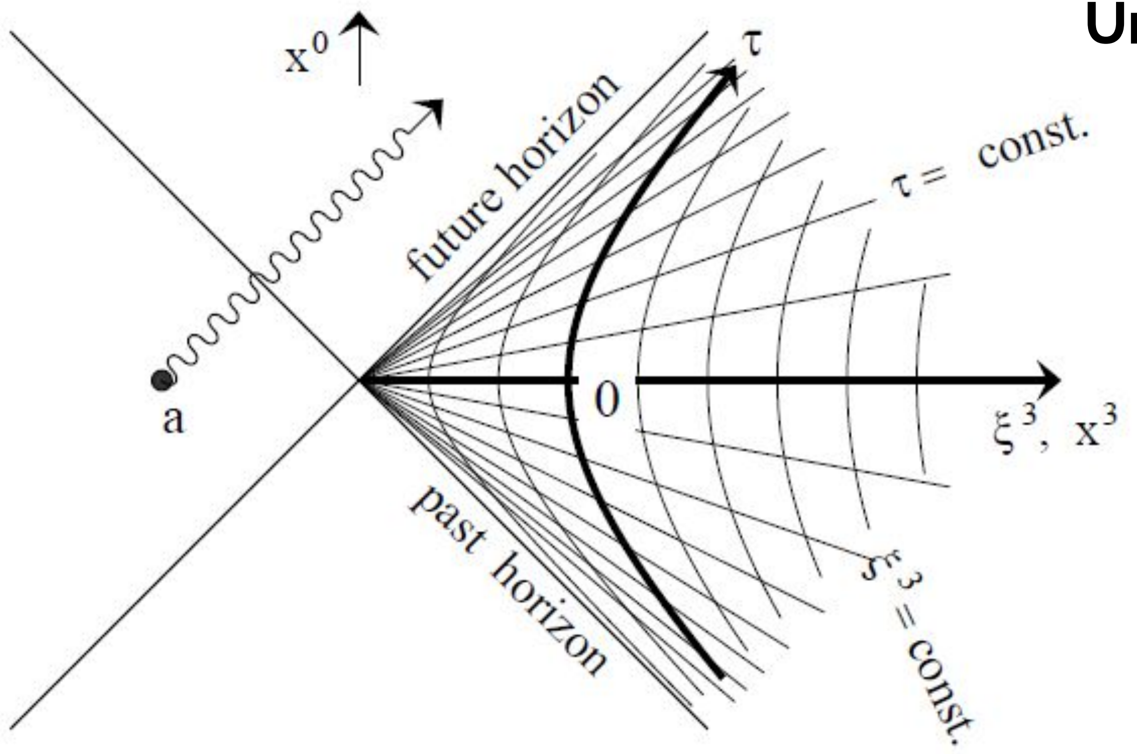
2301.07478

Introduction

Space-time with event horizon

Thermo-dynamics

Unruh effect



Unruh temperature:

$$T_U = \frac{a}{2\pi}$$

https://rojefferson.blog/2021/01/15/qft-in-curved-space-part-2-bogolyubov-transformations-the-unruh-effect/

Unruh effect

Two level detector:

$$x^0(\tau) = \sinh\tau, \qquad x^1(\tau) = \cosh\tau,$$

$$\hat{H}_0 = -\Omega|g\rangle\langle g| + \Omega|e\rangle\langle e|$$

$$\hat{H}_{int}(\tau) = g\chi(\tau)\hat{\varphi}(x^\mu(\tau))[|e\rangle\langle g|e^{i\Omega\tau} + |g\rangle\langle e|e^{-i\Omega\tau}].$$

$$P_{|0_M\rangle}(g\to e) = \frac{g^2}{2\pi}\int_{-\infty}^{\infty} d\delta_\tau \int_{-\infty}^{\infty} d\tau\,\chi(\tau)\chi(\tau+\delta_\tau)e^{-i\Omega\delta_\tau}K_0\big(m\sqrt{2-2\cosh(\delta_\tau - i0)}\big).$$

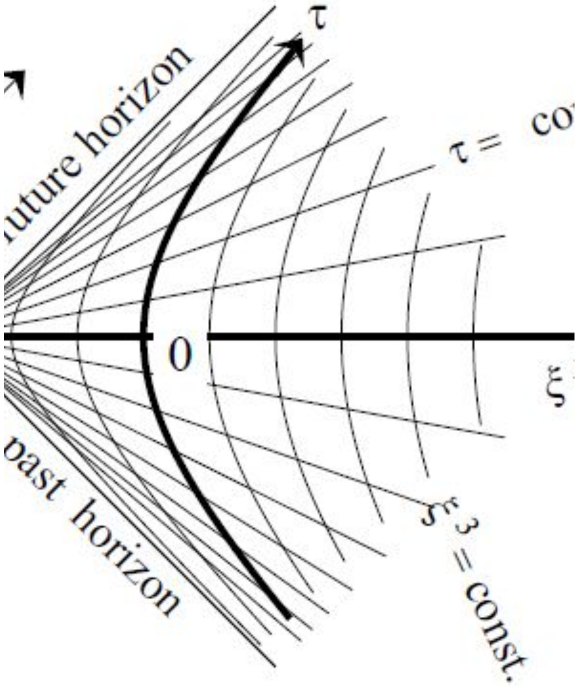
$$P(e\to g) \propto e^{\pi\Omega}, \quad P(g\to e) \propto e^{-\pi\Omega}$$

$$\dot{n}_g = P(e\to g)n_e - P(g\to e)n_g$$

$$\dot{n}_e = P(g\to e)n_g - P(e\to g)n_e$$

$$n_e/n_g = e^{-2\pi\Omega}$$

$$T_U = \frac{1}{2\pi}$$



Fundamental questions?

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left(\partial_\mu \varphi \partial_\mu \varphi - m^2 \varphi^2 \right)$$

$$f(r_0) = 0$$

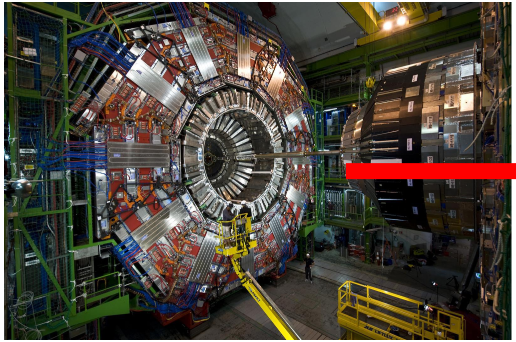
$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2 \left(\sin^2 \theta d\varphi^2 + \theta^2 \right),$$

- **Self-Interacting theory**
- **How to define quantum states?**

Thermal properties:

- **Debye screening**
- **Energy and pressure density**
- **Entropy, Free energy**

Accelerating frame



a

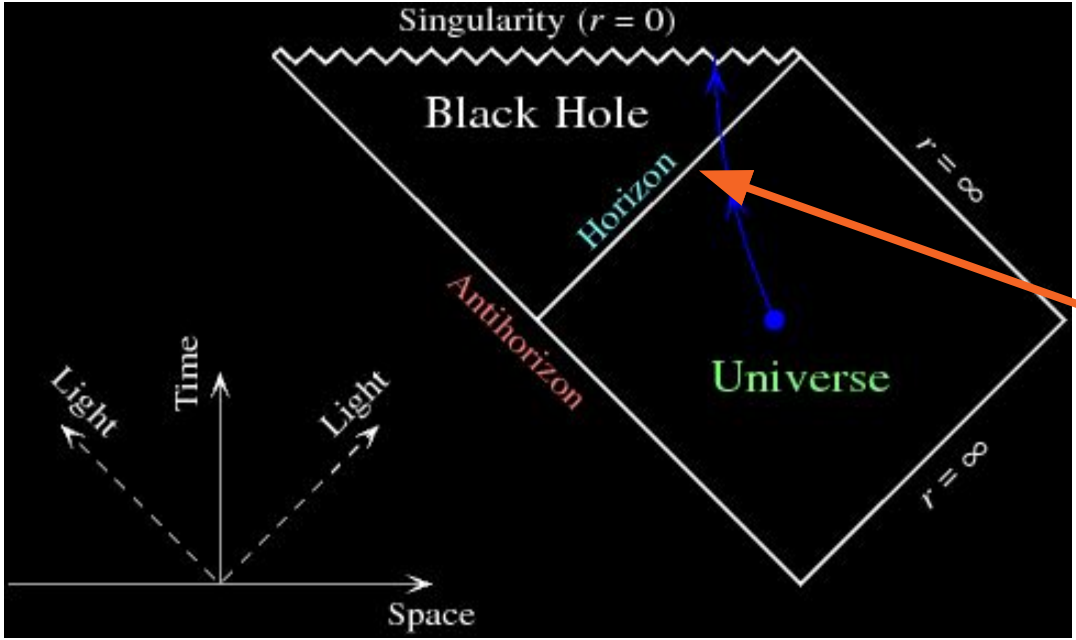
non-inertial!

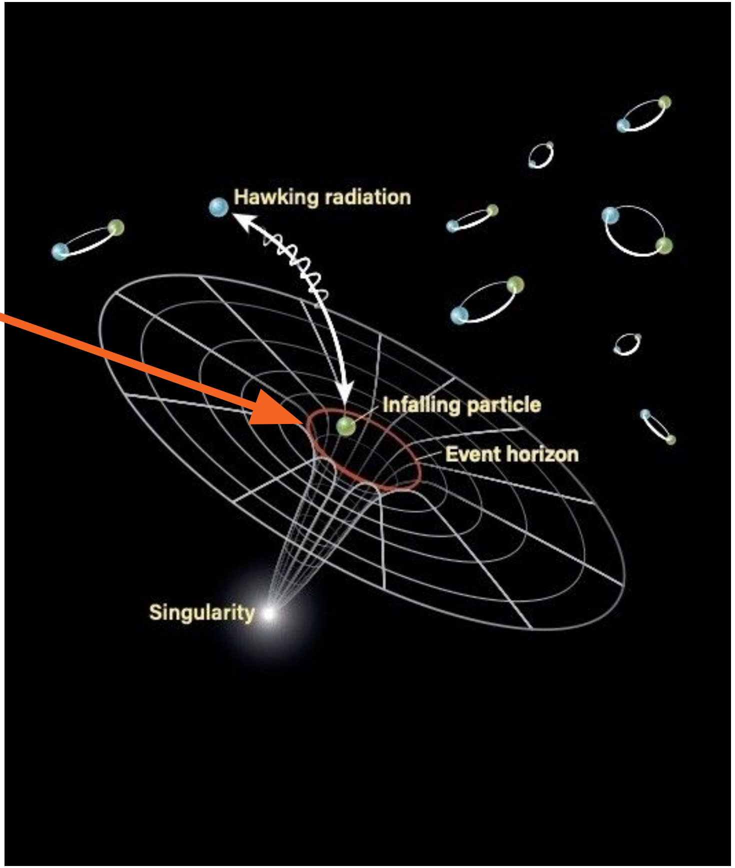
Minkowski Vacuum equals to thermal bath

$$T_U = \frac{a}{2\pi}$$

$$2.47 * 10^{20} m/s^2 \rightarrow 1K$$

Hawking effect



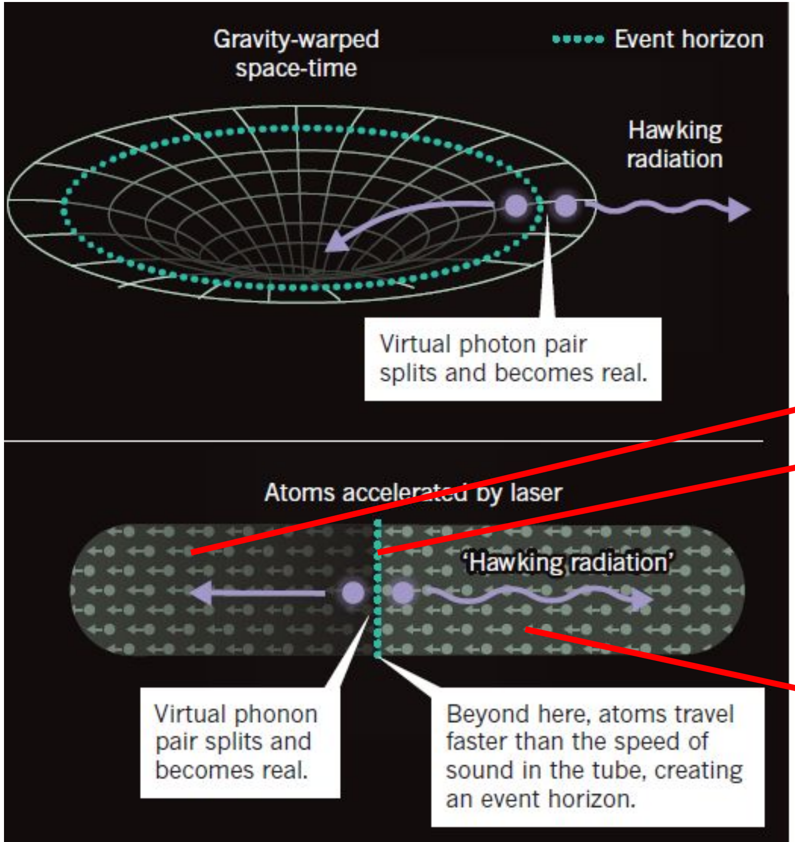


$$ds^2 = \left(1 - \frac{r_s}{r}\right)dt^2 - \frac{dr^2}{1 - \frac{r_s}{r}} - r^2\left(\sin^2\theta d\varphi^2 + \theta^2\right), \quad r_s = 2MG.$$

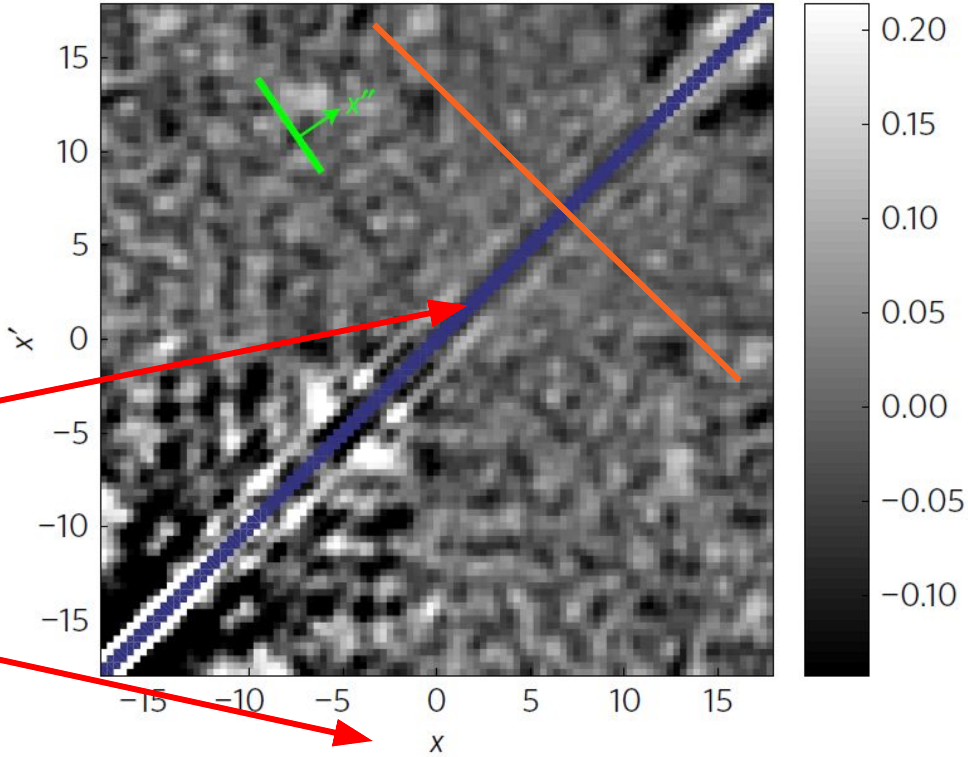
$$T_H = \frac{1}{8\pi GM}$$

Thermal effects

Steinhauer, J. Observation of quantum Hawking radiation and its entanglement in an analogue black hole. *Nature Phys* 12, 959–965 (2016)



Fourier transformation



Quantum states

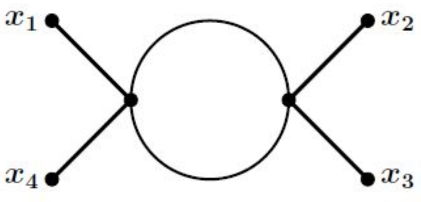
Quantum states

$$\langle\hat{O}\rangle = \mathrm{Tr}\hat{\rho}\hat{O}, \quad \hat{\rho} = ?$$

$$\hat{\varphi} = \sum_i f_i \hat{a}_i + f_i^* \hat{a}_i^\dagger +$$

$$\langle \hat{a}_i^\dagger \hat{a}_j \rangle = \delta_{ij} n(E_i)$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4!}\varphi^4$$



Minkowski space-time!

$$\dot{n}(E_i) \propto \lambda^2 \sum_{jkl} \#_{jkl}\Big[(1+n(E_i))(1+n(E_j))n(E_k)n(E_l) - n(E_i)n(E_j)(1+n(E_k))(1+n(E_l))\Big]$$

$$\rightarrow \quad n(E) = \frac{1}{e^{E/T}-1}$$

Quantum states

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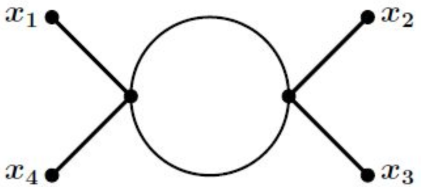


Minkowski space-time!

In any space-time with time-like Killing vector

$$\dot{n}(E_i) \propto \lambda^2 \sum_{jkl} \#_{jkl} \Big[(1 + n(E_i))(1 + n(E_j)) n(E_k) n(E_l) - \\ - n(E_i) n(E_j)(1 + n(E_k))(1 + n(E_l)) \Big]$$

$$\rightarrow \quad n(E) = \frac{1}{e^{E/T} - 1}$$

Quantum states

$$\langle \hat{O} \rangle = \mathrm{Tr}\hat{\rho}\hat{O}, \quad \hat{\rho} = ?$$

$$\hat{\varphi} = \sum_i f_i \hat{a}_i + f_i^* \hat{a}_i^\dagger +$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4!}\varphi^4$$

$$\langle \hat{a}_i^\dagger \hat{a}_j \rangle = \delta_{ij} \frac{1}{e^{E_i/T} - 1}$$

$$\hat{\rho} = e^{-\beta \hat{H}}/Z, \quad \beta \equiv \frac{1}{T}$$

On static space-times with horizons

Fundamental questions?

$$S = \frac{1}{2}\int d^4x\,\sqrt{-g}\left(\partial_\mu\varphi\partial_\mu\varphi - m^2\varphi^2\right)$$

$$f(r_0) = 0$$

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2\left(\sin^2\theta d\varphi^2 + \theta^2\right),$$

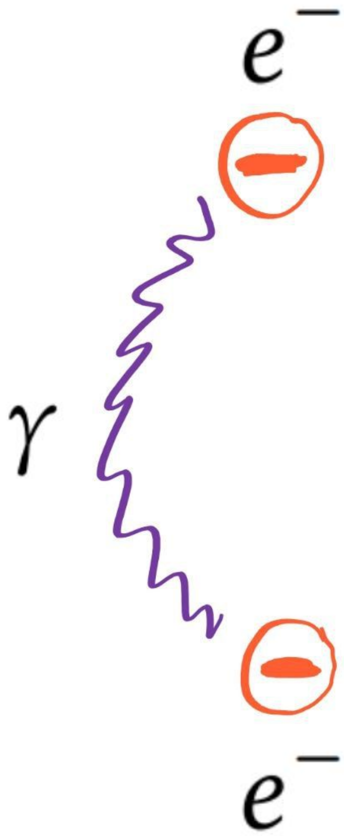
- Self-Interacting theory
- How to define quantum states?

Thermal properties:

- Debye screening
- Energy and pressure density
- Entropy, Free energy

Debye mass

Debye mass



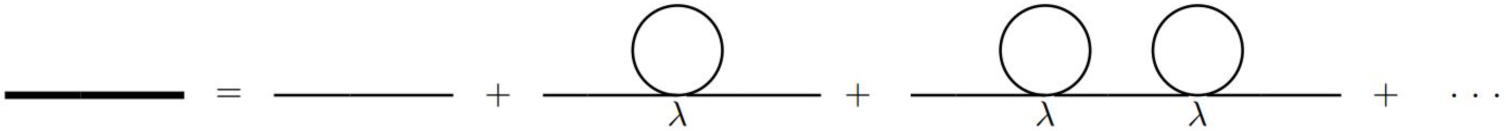
$$V(r) = -e^2 e^{-m_D r}/r$$

$$m_D^2 = e^2 T^2/3$$

Debye mass in flat space-time (perturbative)

$$Z = \int D\phi \; exp\left[-\int_0^\beta d\tau \int d^3x \left(-\frac{1}{2}\partial_\mu \phi(X)\partial^\mu \phi(X) + \frac{\lambda}{4!}\phi^4(X) + \zeta R \phi^2(X) \right) \right]$$

$$\langle T\phi(X_1)\phi(X_2)\rangle\Big|_{\lambda=0} = G_{\lambda^0}(X_1, X_2) \equiv \frac{1}{\beta}\sum_{\omega_n}\int \frac{d^3p}{(2\pi)^3}\frac{e^{iP\cdot(X_1-X_2)}}{-P^2}.$$

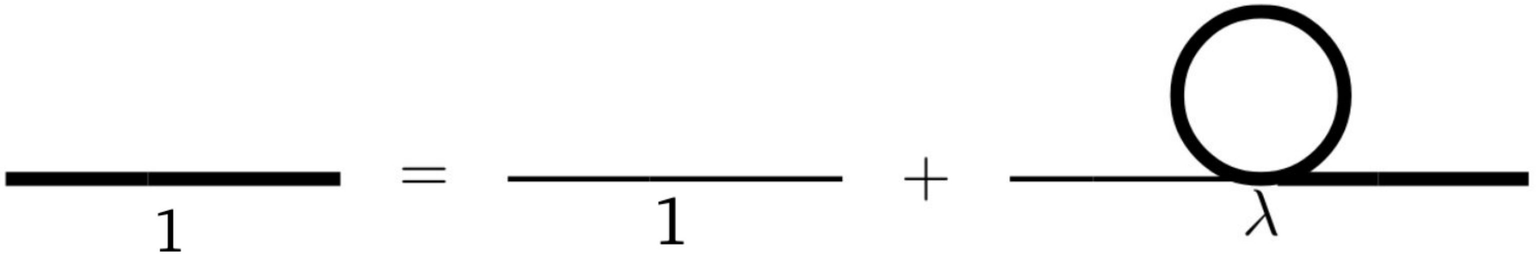


$$G_{\lambda^1}(X_1, X_2) = \frac{1}{\beta}\sum_{\omega_n}\int \frac{d^3p}{(2\pi)^3}\frac{e^{iP\cdot(X_1-X_2)}}{-P^2 + m^2_{\lambda^1}},$$

$$m^2_{\lambda^1} = \frac{\lambda}{2}\int \frac{d^3k}{(2\pi)^3}\frac{1}{|k|}\frac{1}{e^{\beta|k|}-1} = \frac{\lambda}{24}\frac{1}{\beta^2}$$

Debye mass in flat space-time

$$Z = \int D\phi \; exp\left[-\int_0^\beta d\tau \int d^3x \left(-\frac{1}{2}\partial_\mu \phi(X)\partial^\mu \phi(X) + \frac{\lambda}{4!}\phi^4(X) + \zeta R\phi^2(X) \right) \right]$$



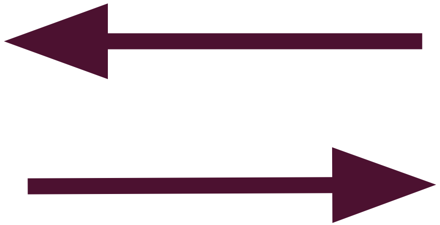
$$\frac{1}{P^2 - m^2_{thermal}} \qquad \frac{1}{P^2}$$

$$\beta = \frac{1}{T}$$

$$m^2_{thermal} = \frac{1}{\beta^2}\left(\frac{\lambda}{24} - \frac{\lambda^{3/2}}{16\sqrt{6}\pi} \right) + O(\lambda^2).$$

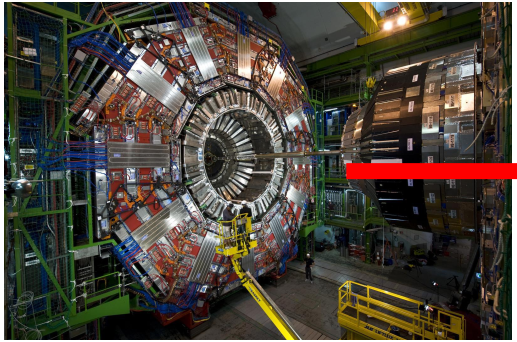
Debye mass in space-times with horizons

Space-time with event horizon



Thermo-dynamics

Accelerating frame



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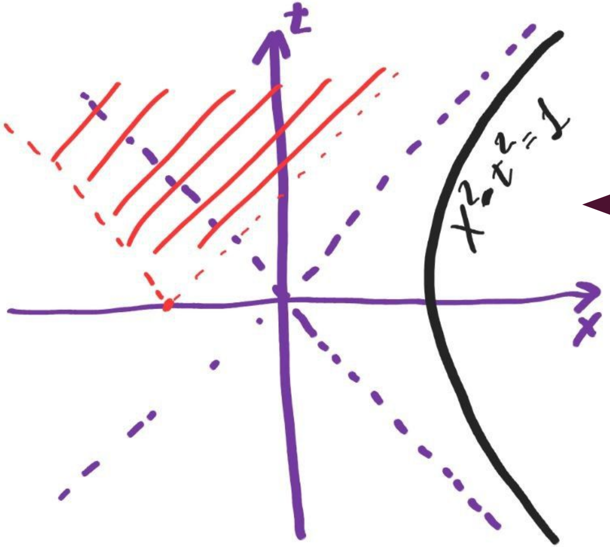


$$ds^2 = e^{2\xi\alpha}\left(d\eta^2 - d\xi^2\right) - d\vec{z}_\perp^2$$

EOM: $\left(\partial_\eta^2 - \partial_\xi^2 - e^{2\xi}\partial_{\vec{z}}^2\right)\varphi(\eta, \xi, \vec{z}) = 0$

What about physical mass?

Green function with physical mass: $S_m = \int d^4x \frac{m^2\varphi^2}{2}$

$$\int_0^{+\infty} \frac{d^2k d\omega}{(2\pi)^2\pi^2} \sinh(\pi\omega) \frac{1}{e^{\beta\omega}-1} e^{i\omega(\eta_1-\eta_2)} e^{i\vec{k}(\vec{z}_1-\vec{z}_2)} K_{i\omega}\left(\sqrt{m^2+k^2}e^{\xi_1}\right) K_{i\omega}\left(\sqrt{m^2+k^2}e^{\xi_2}\right) +$$

$$+ \int_0^{+\infty} \frac{d^2k d\omega}{(2\pi)^2\pi^2} \sinh(\pi\omega) \left[1 + \frac{1}{e^{\beta\omega}-1}\right] e^{-i\omega(\eta_1-\eta_2)} e^{i\vec{k}(\vec{z}_1-\vec{z}_2)} K_{i\omega}\left(\sqrt{m^2+k^2}e^{\xi_1}\right) K_{i\omega}\left(\sqrt{m^2+k^2}e^{\xi_2}\right).$$

The spectrum remains massless!

$$:\hat{H}: = \int_0^{\infty} d\omega\,\omega \int d^2k\, \hat{a}^{\dagger}_{\omega,\vec{k}} \hat{a}_{\omega,\vec{k}}$$

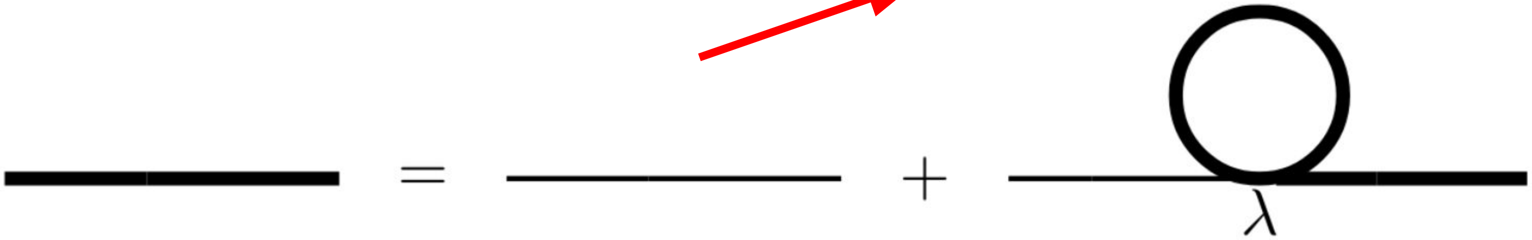
Accelerating frame

Bare Green function:

$$G_0(X_1,X_2) = \frac{1}{\beta}\sum_{\omega_n}\int\frac{d^2\vec{k}}{(2\pi)^2}\int_0^\infty\frac{d\omega}{\pi^2}\frac{2\omega\sinh\pi\omega}{\omega_n^2+\omega^2}e^{-i\omega_n(\eta_2-\eta_1)}e^{i\vec{k}(\vec{z}_2-\vec{z}_1)}K_{i\omega}(ke^{\xi_1})K_{i\omega}(ke^{\xi_2})$$

Dimensional regularization:

$$D = 4 - 2\epsilon$$



Dressed Green function:

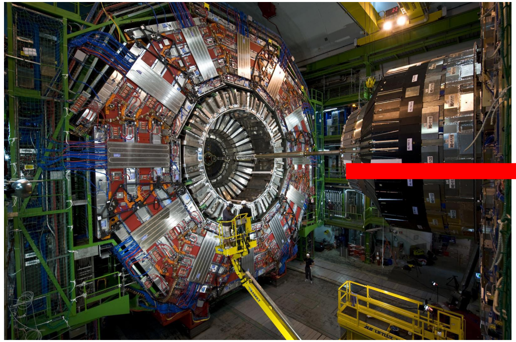
$$G_{\lambda^1}(X_1,X_2) = \frac{1}{\beta}\sum_{\omega_n}\int\frac{d^2\vec{k}}{(2\pi)^2}\int_0^\infty\frac{d\omega}{\pi^2}\frac{2\omega\sinh\pi\omega}{\omega_n^2+\omega^2+M_{\lambda^1}^2}e^{-i\omega_n(\eta_2-\eta_1)}e^{i\vec{k}(\vec{z}_2-\vec{z}_1)}K_{i\omega}(ke^{\xi_1})K_{i\omega}(ke^{\xi_2})$$

Debye mass:

$$M_{\lambda^1}^2 = \frac{\lambda}{24}\left(\frac{1}{\beta^2}-\frac{1}{(2\pi)^2}\right)$$

$$\beta = \beta_U = 2\pi \rightarrow M_{\lambda^1}^2 = 0$$

Accelerating frame



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Minkowski Vacuum equals to thermal bath

$$T_U = \frac{a}{2\pi}$$

So accelerated observers see the vacuum as the thermal bath with the Unruh temperature. But this thermal bath does not have the Debye screening effect!

Accelerating frame

$$G_{\lambda^1}(X_1, X_2) = \int \frac{d^2\vec{k}}{(2\pi)^2} \int_0^\infty \frac{d\omega}{\pi^2} \frac{\omega \sinh \pi\omega}{E_\omega} e^{i\vec{k}(\vec{z}_2 - \vec{z}_1)} K_{i\omega}\left(ke^{\xi_1}\right) K_{i\omega}\left(ke^{\xi_2}\right) \times$$
$$\times \left[e^{-E_\omega(\eta_2 - \eta_1)} \left(1 + \frac{1}{e^{\beta E_\omega} - 1}\right) + e^{E_\omega(\eta_2 - \eta_1)} \frac{1}{e^{\beta E_\omega} - 1} \right],$$

Debye mass:

$$M^2_{\lambda^1} = \frac{\lambda}{24}\left(\frac{1}{\beta^2} - \frac{1}{(2\pi)^2}\right)$$

Spectrum:

$$E_\omega = \sqrt{\omega^2 + M^2_{\lambda^1}}$$

What about physical mass?

In Minkowski space-time thermal and physical mass play the same role

In accelerated frame thermal and physical mass play the different roles

	Physical mass M_{phys}	Thermal mass $M_{thermal}$
Minkowski	$\frac{1}{-P^2} \to \frac{1}{-P^2+M_{phys}^2}$	$\frac{1}{-P^2} \to \frac{1}{-P^2+M_{thermal}^2}$
Rindler	$K_{i\omega}\left(ke^{\xi_2}\right) \to K_{i\omega}\left(\sqrt{k^2+M_{phys}^2}e^{\xi_2}\right)$	$\frac{1}{\omega_n^2+\omega^2} \to \frac{1}{\omega_n^2+\omega^2+M_{thermal}^2}$

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left(\partial_\mu \varphi \partial_\mu \varphi - m^2 \varphi^2 \right)$$

$$f(r_0) = 0$$

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2 \left(\sin^2 \theta d\varphi^2 + \theta^2 \right),$$

- Self-Interacting theory
- How to define quantum states?

Thermal properties:

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- Energy and pressure density
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Energy density

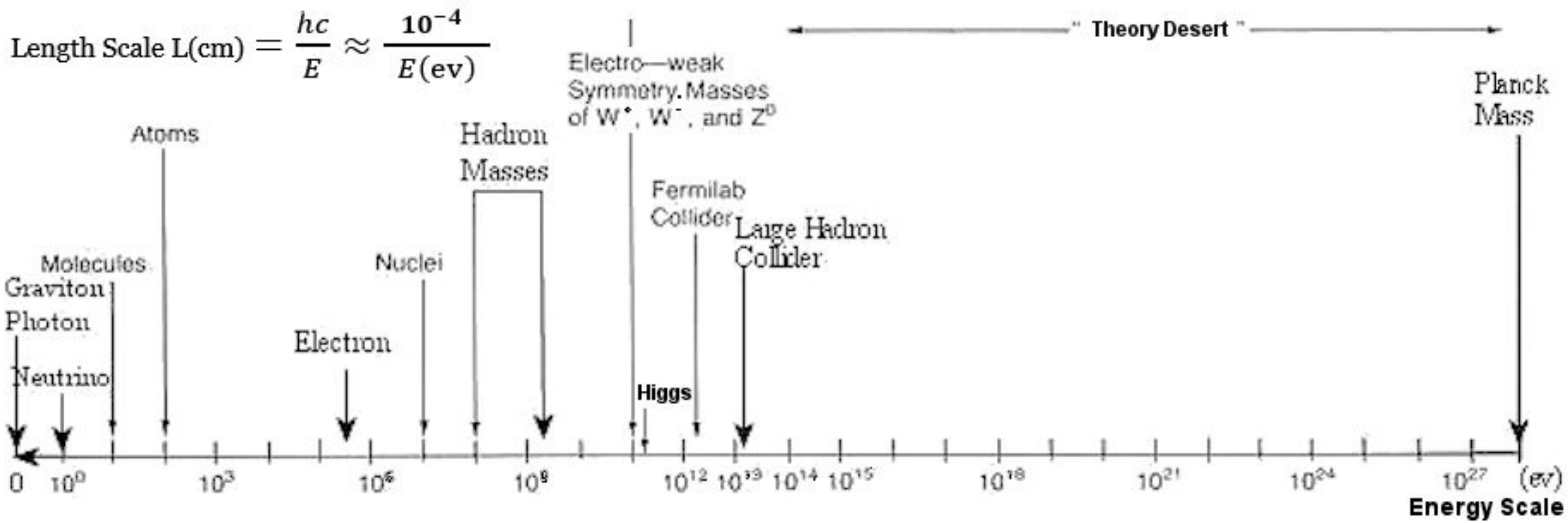
$$ds^2 = e^{2\xi}(d\eta^2 - d\xi^2) - d\vec{z}^2$$

$$Z = \int D\phi \; exp\left[- \int_0^\beta d\tau \int d^3x \left(-\frac{1}{2}\partial_\mu \phi(X)\partial^\mu \phi(X) + \frac{\lambda}{4!}\phi^4(X) + \zeta R \phi^2(X) \right) \right]$$

$$\langle T^\mu_\nu \rangle^{reg} = e^{-4\xi}\left[\frac{\pi^2}{90}\left(\frac{1}{\beta^4} - \frac{1}{(2\pi)^4} \right) - \frac{1}{48}\frac{\lambda}{4!}\left(\frac{1}{\beta^2} - \frac{1}{(2\pi)^2} \right)^2 + \right.$$
$$\left. + \frac{1}{12\pi}\left(\frac{\lambda}{4!} \right)^{3/2} \frac{1}{\beta}\left(\frac{1}{\beta^2} - \frac{1}{(2\pi)^2} \right)^{\frac{3}{2}} \right] \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + O(\lambda^2).$$

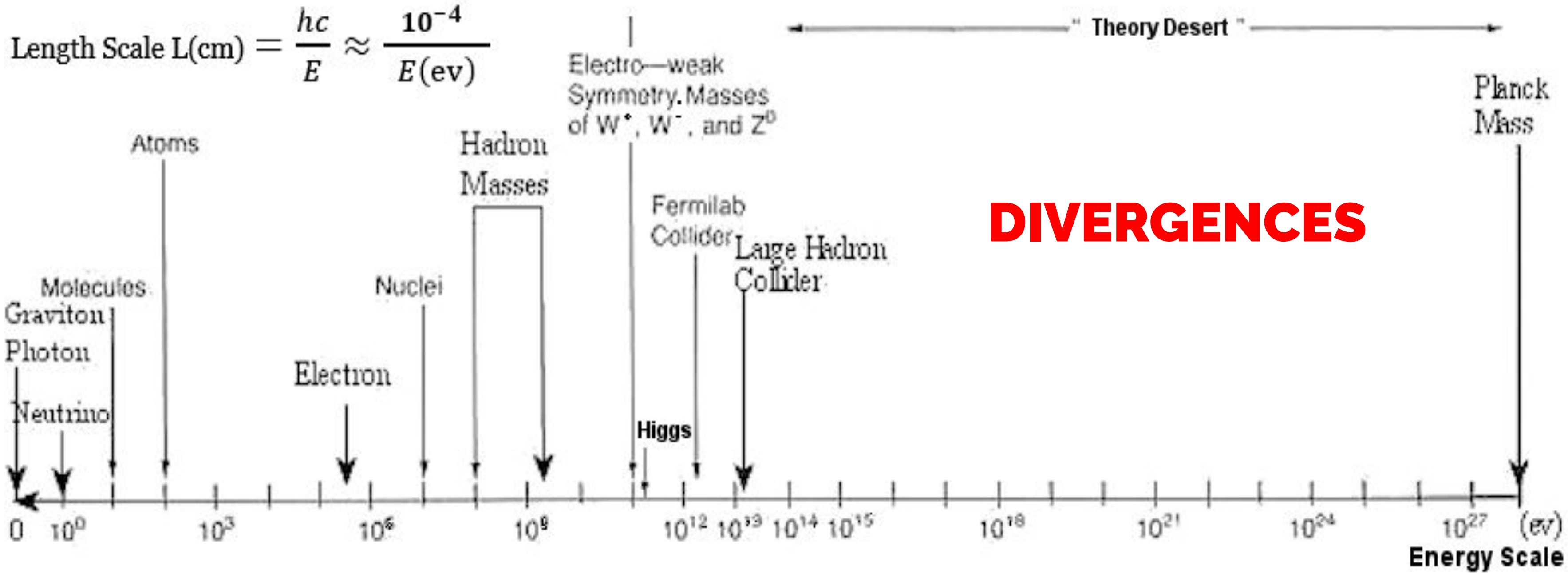
UV - IR decoupling

UV - IR decoupling



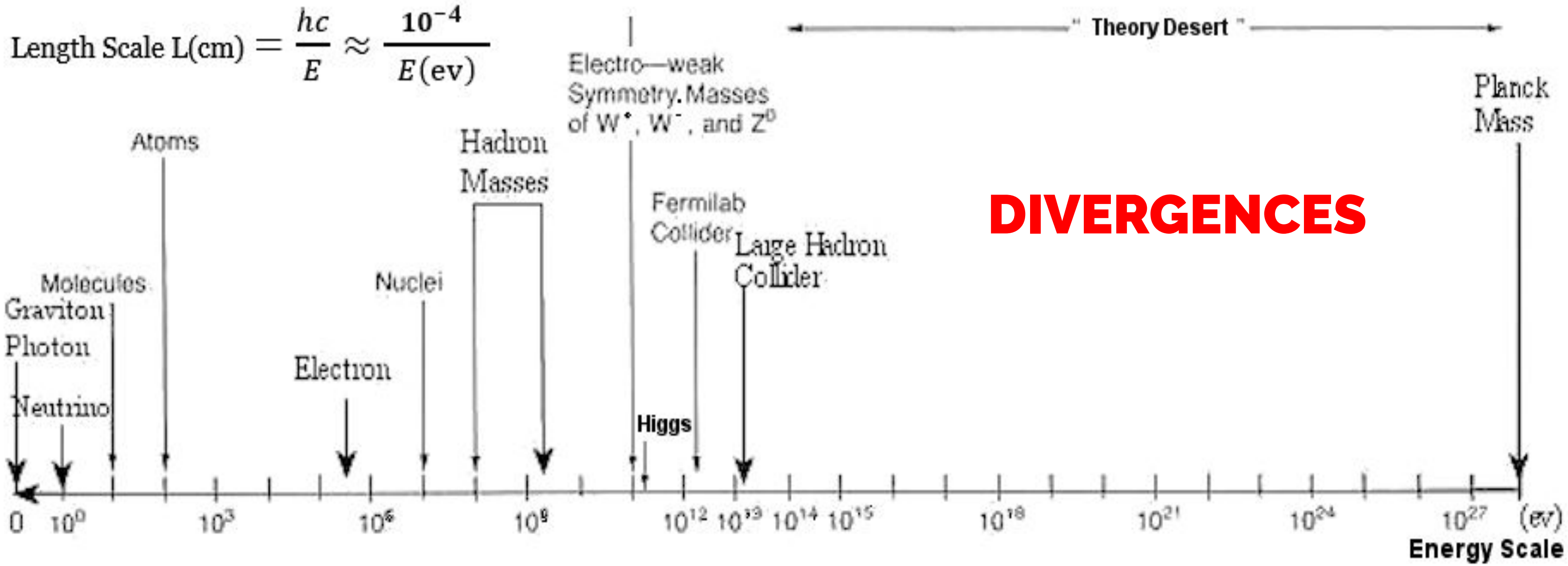
UV - IR decoupling

It is usually possible to organize physical phenomena according to the energy scale or distance scale. The theory of renormalization group is based on this paradigm. The short-distance, ultraviolet (UV) physics does not directly affect qualitative features of the long-distance, infrared (IR) physics, and vice versa



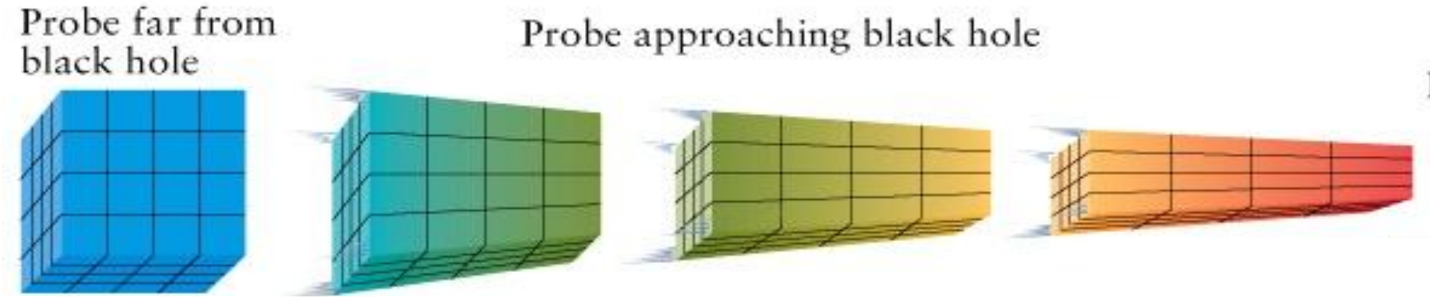
UV - IR decoupling

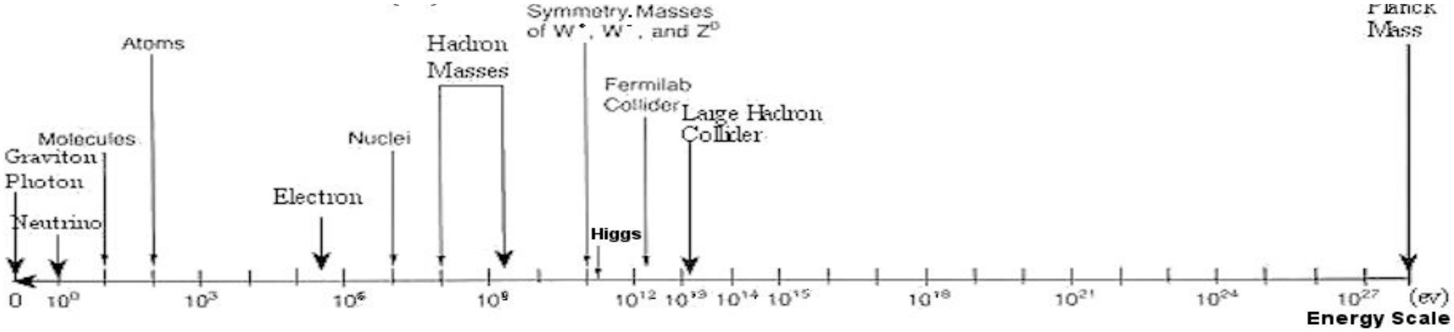
One can somehow regularize the theory to avoid UV divergences and obtain effective IR theory.



UV - IR decoupling

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1}dr^2 - r^2 d\Omega^2$$





UV - IR decoupling

This is why there black holes break UV - IR decoupling

$$S_m = \int d^4x \frac{m^2\varphi^2}{2} \longrightarrow \, : \hat{H} : = \int_0^\infty d\omega\omega \int d^2k \hat{a}^\dagger_{\omega,\vec{k}} \hat{a}_{\omega,\vec{k}}$$

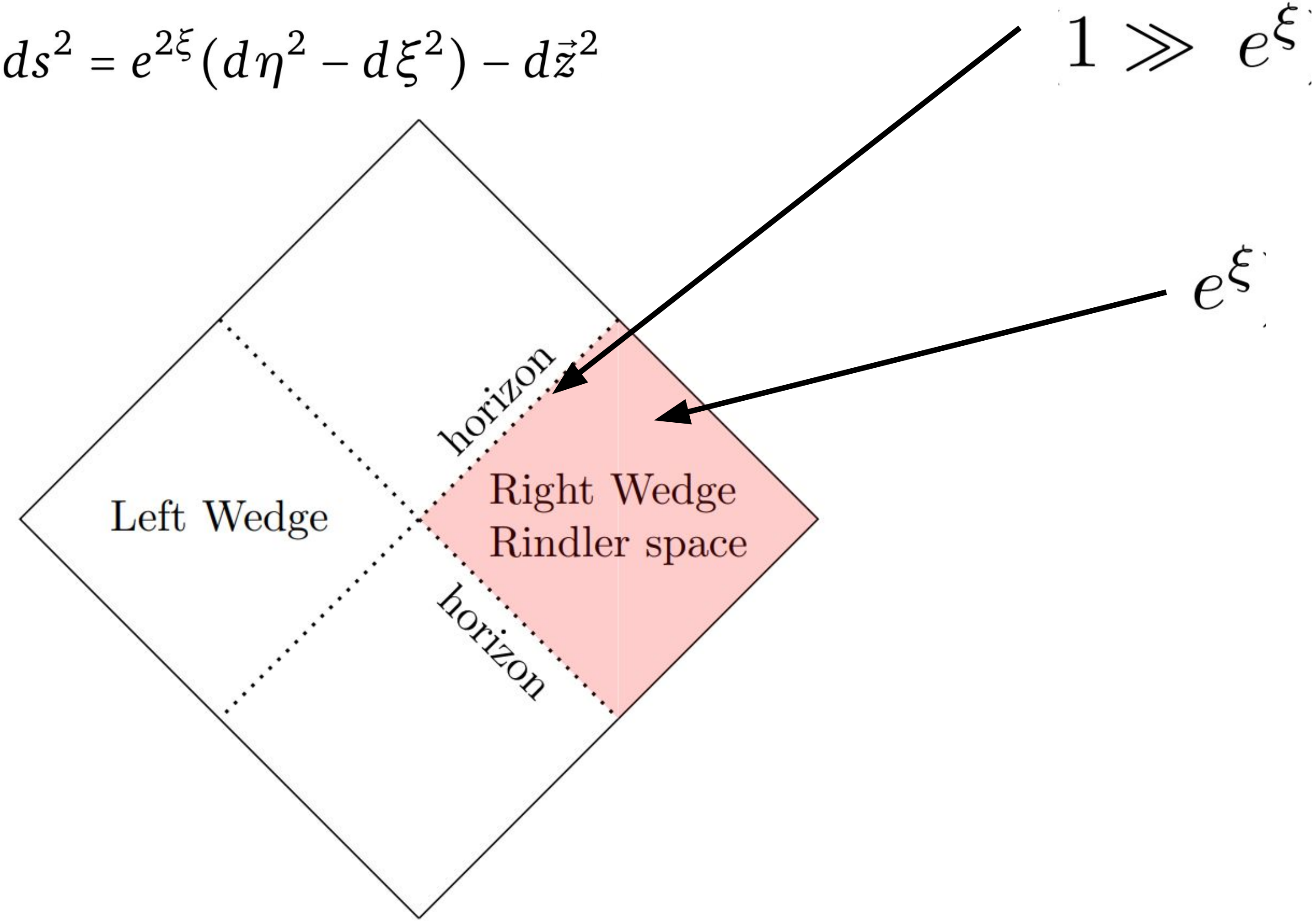
But there is solution!

Debye mass

$$\longrightarrow E_\omega = \sqrt{\omega^2 + M^2_{\lambda^1}}$$

THE END

$$ds^2 = e^{2\xi}(d\eta^2 - d\xi^2) - d\vec{z}^2$$

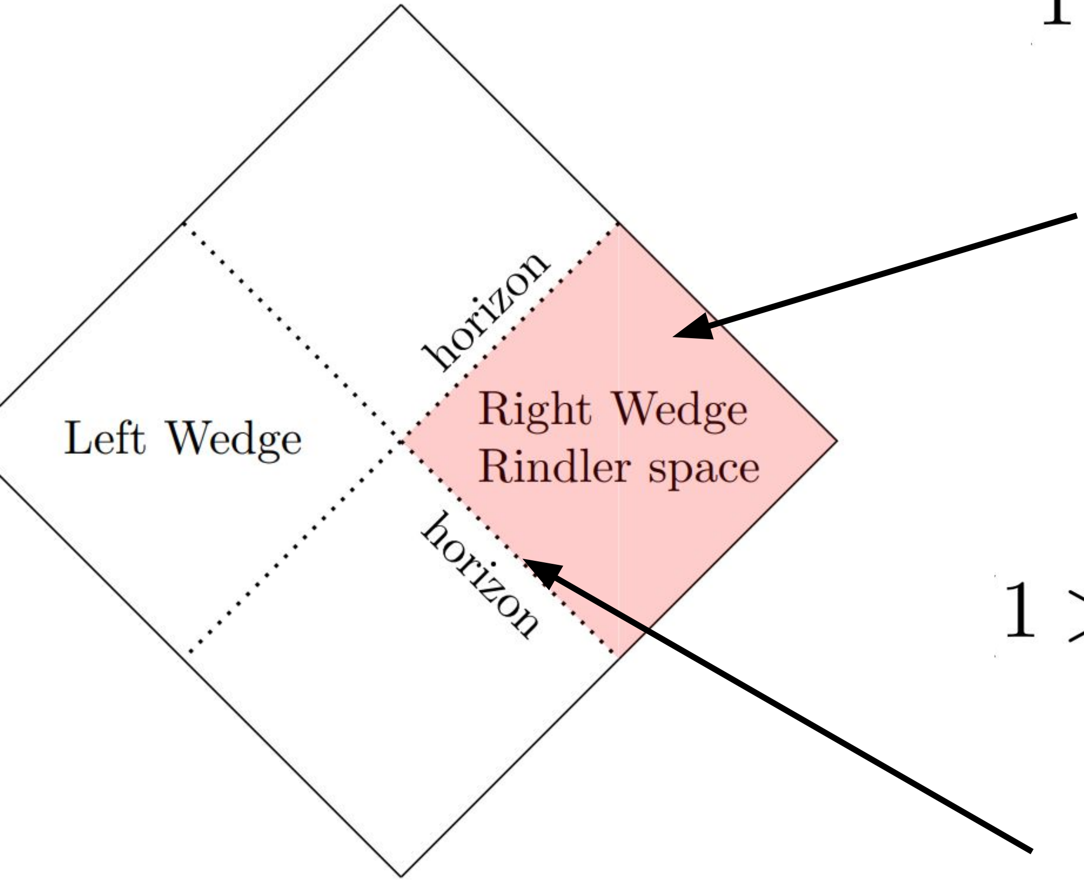


$$1 \gg |\vec{z}_1 - \vec{z}_2|$$

$$W_\beta(\eta_1, \vec{z}_1, \xi | \eta_2, \vec{z}_2, \xi) \approx \frac{1}{4\pi^2} \frac{1}{|\vec{z}_1 - \vec{z}_2|^2}$$

$$1 \gg |\vec{z}_1 - \vec{z}_2| \gg e^{\xi}$$

$$W_\beta(\eta_1, \vec{z}_1, \xi | \eta_2, \vec{z}_2, \xi) \approx \frac{1}{4\pi^2} \frac{1}{|\vec{z}_1 - \vec{z}_2|^2} \frac{2\pi}{\beta},$$



$$1 \gg |\vec{z}_1 - \vec{z}_2|$$

$$\frac{1}{4\pi^2}\frac{1}{|\vec{z}_1 - \vec{z}_2|^2}$$

$$1 \gg |\vec{z}_1 - \vec{z}_2| \gg e^{\xi}$$

$$\frac{1}{4\pi^2}\frac{1}{|\vec{z}_1 - \vec{z}_2|^2}\frac{2\pi}{\beta},$$

$$M^2_{\lambda^{3/2}} = \frac{\lambda}{24}\left(\frac{1}{\beta^2} - \frac{1}{(2\pi)^2}\right) - \frac{\lambda^{3/2}}{16\sqrt{6}\pi}\frac{1}{\beta}\sqrt{\frac{1}{\beta^2} - \frac{1}{(2\pi)^2}} + O(\lambda^2)$$

$$\langle T^{\mu}_{\nu}\rangle^{reg} = e^{-4\xi}\Bigg[\frac{\pi^2}{90}\Big(\frac{1}{\beta^4} - \frac{1}{(2\pi)^4}\Big) - \frac{1}{48}\frac{\lambda}{4!}\Big(\frac{1}{\beta^2} - \frac{1}{(2\pi)^2}\Big)^2 + + \frac{1}{12\pi}\left(\frac{\lambda}{4!}\right)^{3/2}\frac{1}{\beta}\Big(\frac{1}{\beta^2} - \frac{1}{(2\pi)^2}\Big)^{\frac{3}{2}}\Bigg]\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + O(\lambda^2).$$

$$\mu^{4-d}\int\frac{d^{d-2}\vec{k}}{(2\pi)^{d-2}}\int_0^\infty\frac{d\omega}{\pi^2}\frac{\omega\sinh\pi\omega}{\sqrt{\omega^2+M^2}}K_{i\omega}(ke^\xi)K_{i\omega}(ke^\xi)\left(\frac{2}{e^{\beta\sqrt{\omega^2+M^2}}-1}-\frac{2}{e^{2\pi\omega}-1}\right)=$$

$$=e^{-2\xi}\left(\frac{1}{12}\left(\frac{1}{\beta^2}-\frac{1}{4\pi^2}\right)+M\left(\frac{1}{4\pi^3}-\frac{1}{4\beta\pi}\right)+M^2\frac{1-\log\left(\frac{\beta}{2\pi}\right)}{8\pi^2}-\frac{M^3}{18\pi}+O(M^4)\right)$$

$$\mu^{4-d}\int\frac{d^{d-2}\vec{k}}{(2\pi)^{d-2}}\int_0^\infty\frac{d\omega}{\pi^2}\frac{\omega\cosh\pi\omega}{\sqrt{\omega^2+M^2}}K_{i\omega}(ke^\xi)K_{i\omega}(ke^\xi)= \tag{B.11}$$

$$=\mu^{4-d}e^{(2-d)\xi}2^{1-d}\pi^{\frac{2-d}{2}}\cos\left(\frac{\pi d}{2}\right)\Gamma\left(\frac{d-2}{2}\right)\int_0^\infty d\rho\sinh^{2-d}(\rho)\Big[\delta(\rho)+M\Big(I_1(2\rho M)-\boldsymbol{L}_{-1}(2\rho M)\Big)\Big],$$

$$\cosh\pi\omega K_{i\omega}(ke^\xi)K_{i\omega}(ke^\xi)=-\pi\int_0^\infty d\rho\ Y_0\left(2ke^\xi\sinh\rho\right)\cos(2\omega\rho).$$

$$\mu^{4-d}e^{(2-d)\xi}2^{1-d}\pi^{\frac{2-d}{2}}\cos\left(\frac{\pi d}{2}\right)\Gamma\left(\frac{d-2}{2}\right)\int_0^\infty d\rho\sinh^{2-d}(\rho)\Big[\delta(\rho)+M\Big(I_1(2\rho M)-\boldsymbol{L}_{-1}(2\rho M)\Big)\Big]=$$

$$=e^{-2\xi}\left[-\frac{M}{4\pi^3}+M^2\left(\frac{1}{8(-4+d)\pi^2}-\frac{2+3\gamma+\log(\pi)+2\log(\mu)+2\xi}{16\pi^2}\right)+\frac{M^3}{18\pi}\right]+O(M^4). \tag{B.12}$$

$$m^2 = \frac{\lambda}{16\pi^2}\frac{m^2}{d-4} + \frac{\lambda m^2}{32\pi^2}\log\left(\frac{e^{-1+\gamma}m^2}{4\pi\mu^2}\right) + \frac{\lambda}{4\pi^2}\int_0^\infty dp\,\frac{p^2}{\sqrt{p^2+m^2}}\frac{1}{e^{\sqrt{p^2+m^2}}-1}$$

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